SOS3003
Applied data analysis for social science
Lecture note 01-2010

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History

- In the history of civilization there are 2 unrivalled accelerators:
  - The invention of writing about 5-6000 years ago
  - The invention of the scientific method for separating facts from fantasy about 5-600 years ago
- There is no topic more important to learn than the basics of the scientific method
- That does not mean that it is not – at times – rather boring ….

Basics of causal beliefs

- First: doubt what you believe is a causal link until you can give good valid reasons justifying your belief
- Second: there are usually many types of good valid reasons for believing in a particular causal link, for example scientific consensus
  - If the overwhelming majority of certified scientists says that human activities **contribute** to global warming, then we are justified believing that by changing our activities we could contribute less to global warming
- Third: random conjunctures ("correlation") are not good valid reasons for believing in a causal link

Causal correlations

- This class will focus on how to distinguish between random conjunctures and that which might be a valid causal correlation
- That which might be a valid causal correlation will need a *causal mechanism* explaining how the cause can produce the effect before we have a valid reason to believe in the causal link

Causal mechanism

- Elster 2007 *Explaining Social Behaviour*:
- ”mechanisms are frequently occurring and easily recognizable causal patterns that are triggered under generally unknown conditions or with indeterminate consequences” (page 36)
- Also sometimes limited to “causal chains”

Primacy of theory

- To say it more bluntly: If you do not have a believable theory (and this may well start as a fantasy) then regression techniques will tell you nothing even if you find a seemingly non-random correlation
- But without a valid and believable empirical analysis any believable fantasy will remain just that: a fantasy (assuming you cannot find other valid verifications)

Preliminaries

- Prerequisite: SOS1002 or equivalent
- Goal: to read critically research articles from our field of interest
- Required reading
- Term paper: this is part of the examination and evaluation procedure

Goals for the class

- The goal is that each of you shall be able to read critically research articles discussing quantitative data. This means
  - You are to know the pitfalls so you can evaluate the validity of an article
- You are to learn how to perform straightforward analyses of co-variation in ”quantitative” and ”qualitative” data (nominal scale data in regression analysis), and in particular:
  - Also here you have to demonstrate that you know the pitfalls

Required reading SOS3003

- Hamilton, Lawrence C. 1992. *Regression with graphics*. Belmont: Duxbury. Ch 1-8
- Hamilton, Lawrence C. 2008. *A Low-Tech Guide to Causal Modelling*. http://pubpages.unh.edu/~lch/causal2.pdf
- Allison, Paul D. 2002. *Missing Data*. Sage University Paper: QASS 136. London: Sage.

Term paper

- **Deadline for paper: 10 May; delivery by e-mail to <ISSInnlevering@svt.ntnu.no>**
- The term paper shall be an independent work demonstrating how multiple regression can be used to analyze a social science problem. The paper should be written as a journal article, but with more detailed documentation of data and analysis, for example by means of appendices.
- Based on information about the dependent variable a short theoretical discussion of possible causal mechanisms explaining some of the variation in the dependent variable is presented. This leads up to a model formulation and operationalisation of possible causal variables taken from the data set. If missing data on one or more variables causes one or more cases to be dropped from the analysis, the selection problem must be discussed.
- By means of multiple regression (OLS or Logistic) the model should be estimated and the results discussed in relation to the initial theoretical discussion
- More details will be available in a separate paper

Serious errors from the term papers of last fall

- Lack of understanding of varables and measurement scales
  - Relation to measurement units
  - Relation to correlations among variables
  - Relation to dummy coding
- Lack of understanding of measurement units
  - Relation to interpretation of results

Lecture I
Basics of what you are assumed to know

- The following is basically repeating known stuff
- Variable distributions
  - Ringdal Ch 12 p251-270
  - Hamilton Ch 1 p1-23
- Bivariat regression
  - Ringdal Ch 17-18 p361-387
  - Hamilton Ch 2 p29-59

Some basic concepts

- Cause
- Model
- Population
- Sample
- Variable: level of measurement
- Variable: measure of centralization
- Variable: measure of dispersion

Data analysis

- Descriptive use of data
  - Developing classifications
- Analytical use of data
  - Describe phenomena that cannot be observed directly (inference)
  - Causal links between directly eller indirectly observable phenomena (theory or model development)

Causal analysis: from co-variation to causal connection

- From colloquial speach to theory
  - Fantasy and intuition, established science tradition
- From theory to model
  - Operationalisation
- From observation to generalisation
  - Causal analysis

THREE BASIC DIVISIONS

Observed		Real interest
THEORY/ MODEL	-	REALITY
SAMPLE	-	POPULATION
CO-VARIATION	-	CAUSE

On the one hand we have what we are able to observe and record, on the other hand, we have what we would like to discuss and know more about

Basic sources of error

- Errors in theory / model
  - Model specification: valid conclusions require a correct (true) model
- Errors in the sample
  - Selection bias
- Measurement problems
  - Missing cases and measurement errors
  - Validity og reliability
- Multiple comparisons
  - Conclusions are valid only for the sample

From population to sample

- POPULATION (all units)

Simple random sampling

- SAMPLE (selected units)

Unit and variable

- A unit, as a carrier of data, will be contextually defined
  - SUPER - UNIT: e.g. the local community
  - UNIT: e.g. household
  - SUB - UNIT: e.g. person
- Variable: empirical concept used to characterize units under investigation. Each unit is characterized by being given a variable value

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Data matrix and level of measurement

- Matrix defined by Units * Variables
  - A table presenting the characteristics of all investigated units ordered so that all variable values are listed in the same sequence for all units
- Level of measurement for a variable
  - Nominal scale *classification
  - Ordinal scale *classification and rank
  - Interval scale *classification, rank and distance
  - Ratio scale *classification, rank, distance and absolute zero

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Variable analysis

- Description
  - Central tendency and dispersion
  - Form of distribution
  - Frequency distributions and histograms
- Comparing distributions
  - Quantile plots
  - Box plots

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VARIABLE: central tendency

- Mean
  sum of all values of the variable for all units divided by the number of units
- MEDIAN
  The variable value in an ordered distribution that has half the units on each side
- MODUS
  The typical value. The value in a distribution that has the highest frequency

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\sum_{i=1}^{n}(X_i - \bar{X})$$

$$\sum_{i=1}^{n}(X_i - \bar{X})^2 < \sum_{i=1}^{n}(X_i - C)^2$$

$$if$$

$$C \neq \bar{X}$$

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VARIABLE: measures of dispersion I

- MODAL PERCENTAGE
- The percentage of units with value like the mode
- RANGE OF VARIATION
- The difference between highest and lowest value in an ordered distribution
- QUARTILE DIFFERENCE
- Range of variation of the 50% of units closest to the median (Q_3-Q_1)
- MAD - Median Absolute Deviation
- Median of the absolute value of the difference between median and observed value:
  - $MAD(x_i) = \text{median } |x_i - \text{median}(x_i)|$

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VARIABLE: measures of dispersion II

- STANDARD DEVIATION
- Square root of mean squared deviation from the mean
  - $s_y = \sqrt{[(\Sigma_i(Y_i - \tilde{Y})^2)/(n - 1)]}$
- MEAN DEVIATION
- Mean of the absolute value of the deviation from the mean
- VARIANCE
- Standard deviation squared:
  - $s_y^2 = (\Sigma_i(Y_i - \tilde{Y})^2)/(n - 1)$

(nb: here Ỹ is the mean of Y)

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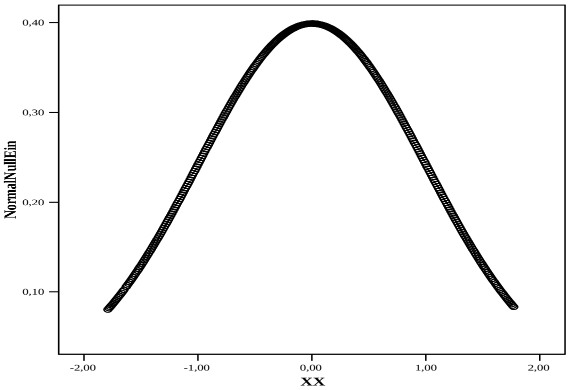
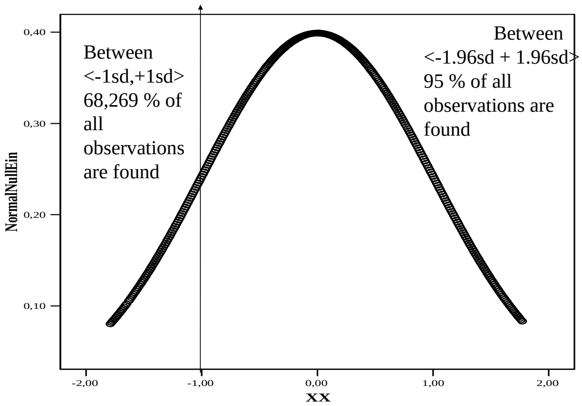
Variable: form of distribution I

- Symmetrical distributions
- Skewed distributions
  - ”Heavy” and ”Light” tails
- Normal distributions
  - Are not ”normal”
  - Are unambiguously determined by mean and variance (μ og σ^2)





Skewed distributions

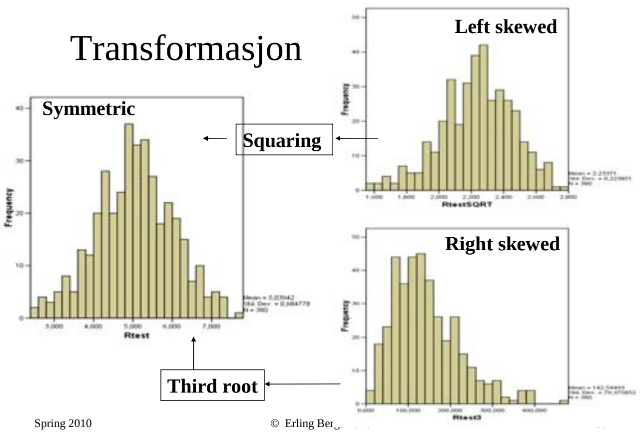
- Positively skewed has $\tilde{Y} > Md$
- Negatively skewed has $\tilde{Y} < Md$
- Symmetric distributions has $\tilde{Y} \approx Md$

- nb: $\tilde{Y}$ = mean of Y

Symmetric distributions

- Median and IQR are resistant against the impact of extreme values
- Mean and standard deviation are not
- In the normal distribution (ND) $s_y \approx IQR/1.35$
- If we in a symmetric distribution find
  - $s_y > IQR/1.35$ then the tails are heavier than in the ND
  - $s_y < IQR/1.35$ then the tails are lighter than in the ND
  - $s_y \approx IQR/1.35$ then the tails are about similar to the ND

Transformasjon



Variable: analyzing distributions I

- Box plot
  - The box is constructed based on the quartile values Q_1 og Q_3 . Observations within < Q_1, Q_3> are in the box-
  - Adjacent large values are defined as those outside the box but inside Q_3 + 1.5*IQR or Q_1 - 1.5*IQR
  - Outliers (seriously extreme values) are those outside of Q_3 + 1.5*IQR or Q_1 - 1.5*IQR

Variables: analyzing distributions II

- Quantiles is a generalisation of quartiles and percentiles
- Quantile values are variable values that correspond to particular fractions of the total sample or observed data, e.g.
  - Median is 0.5 quantile (or 50% percentile)
  - Lower quartile is 0.25 quantile
  - 10% percentile is 0.1 quantile …

Variables: analyzing distributions III

- Quantile plots
  - Quantile values against value of variable
    - The Lorentz curve is a special case of this (it gives us the Gini-index)
- Quantile-Normal plot
  - Plot of quantile values on one variable against quantile values of a Normal distribution with the same mean and standard deviation

Example: Randaberg 1985

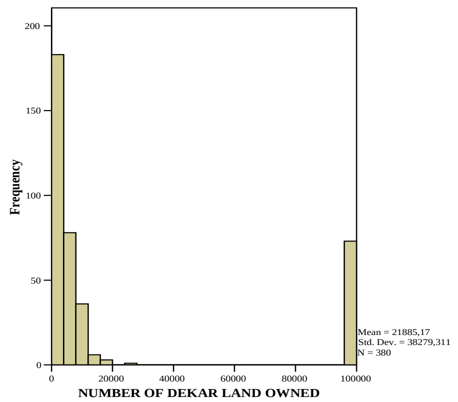
- Questionnaire: (the number of decare land you own / 10 da = 1 ha)

Q: ANTALL DEKAR GRUNN DU eier:_________

(Number of decare you own: _____)

NUMBER OF DEKARE LAND OWNED

	NUMBER OF DEKARE LAND OWNED	Valid N (listwise)
N	380	380
Minimum	0	
Maximum	99900	
Mean	21885.17	
Std. Deviation	38279.311	

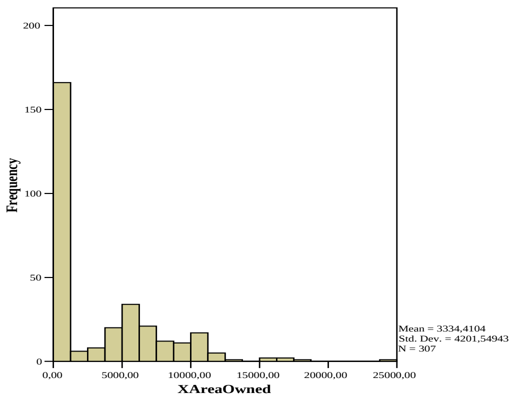


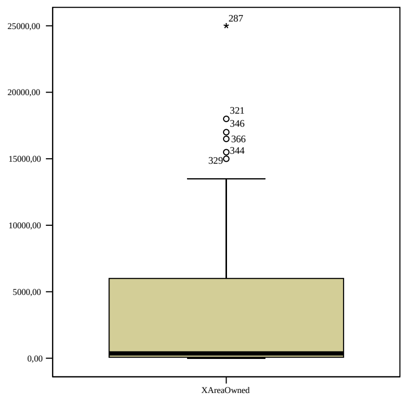
XAreaOwned

(NUMBER OF DEKARE LAND OWNED)

	XAreaOwned	Valid N (listwise)
N	307	307
Minimum	.00	
Maximum	25000.00	
Mean	3334.4104	
Std. Deviation	4201.54943	

		XAreaOwned	Valid N (listwise)
N	Statistic	307	307
Range	Statistic	25000.00	
Minimum	Statistic	.00	
Maximum	Statistic	25000.00	
Sum	Statistic	1023664.00	
Mean	Statistic	3334.4104	
	Std. Error	239.79509	
Std. Deviation	Statistic	4201.54943	
Variance	Statistic	17653017.596	
Skewness	Statistic	1.352	
	Std. Error	.139	
Kurtosis	Statistic	2.194	
	Std. Error	.277	





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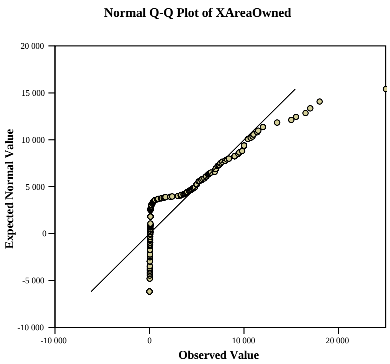
NB

Figures from SPSS are mirrors of figures in Hamilton



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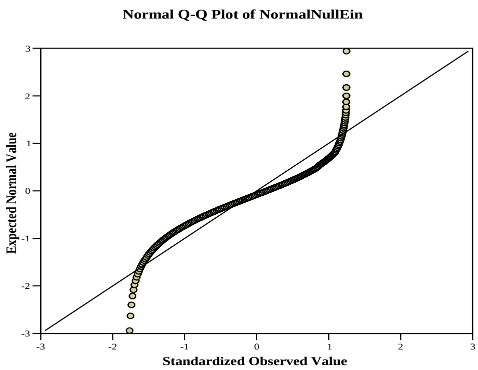

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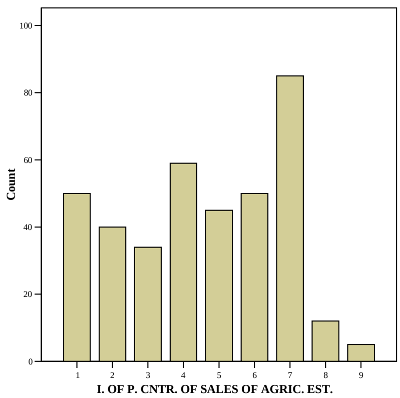
Questionnaire:

- **Hvor viktig er det at myndighetene kontrollerer og regulerer bruken av arealer gjennom for eksempel kontroll av**
- av tomtetildelinger (kommunal formidl.)
  1 2 3 4 5 6 7 8
- avkjørsler fra hus til vei
  1 2 3 4 5 6 7 8
- kjøp og salg av landbrukseiendommer
  1 2 3 4 5 6 7 8

Importance of public control of sales of agric. estates

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	50	13.2	13.2	13.2
	2	40	10.5	10.5	23.7
	3	34	8.9	8.9	32.6
	4	59	15.5	15.5	48.2
	5	45	11.8	11.8	60.0
	6	50	13.2	13.2	73.2
	7	85	22.4	22.4	95.5
	8	12	3.2	3.2	98.7
	9	5	1.3	1.3	100.0
	Total	**380**	**100.0**	**100.0**	



Questionnaire: coding

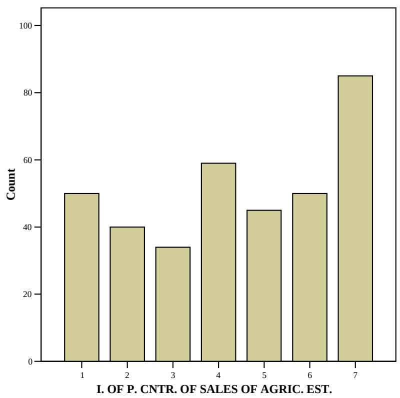
Ved utfylling: **sett ring rundt et tall som synes å gi passelig uttrykk for viktigheten når 1 betyr svært lite viktig og 7 særdeles viktig, eller sett et kryss inne i parantesene ( ) som står bak svaret du velger**
På noen spørsmål kan du krysse av flere svar

	lykkes dårlig/ lite viktig					lykkes godt/ svært viktig	vet ikke
Kodeverdi	1	2	3	4	5 6	7	8

Dei som ikkje kryssar av noko svar vert koda 9 (ie. missing)

I. OF P. CNTR. OF SALES OF AGRIC. EST.

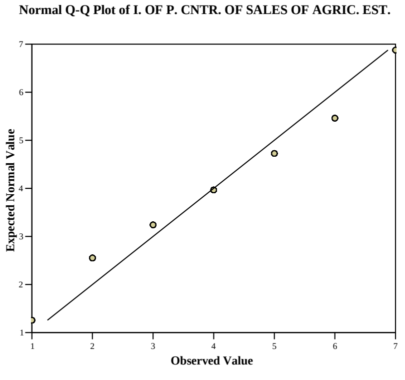
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	50	13.2	13.8	13.8
	2	40	10.5	11.0	24.8
	3	34	8.9	9.4	34.2
	4	59	15.5	16.3	50.4
	5	45	11.8	12.4	62.8
	6	50	13.2	13.8	76.6
	7	85	22.4	23.4	100.0
	Total	**363**	**95.5**	**100.0**	
Missing	8	12	3.2		
	9	5	1.3		
	Total	**17**	**4.5**		
Total		**380**	**100.0**		

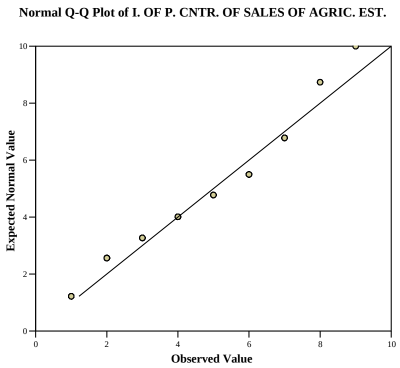


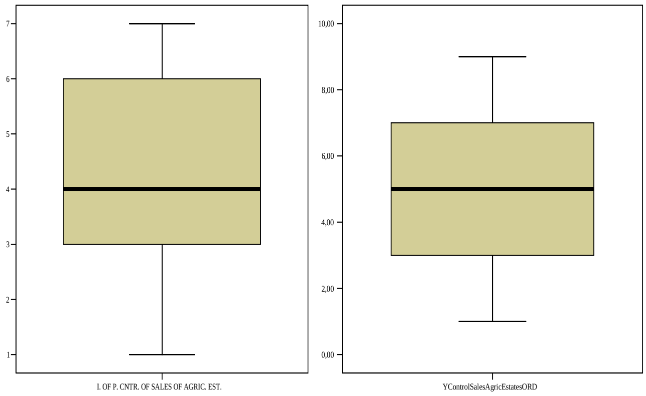
		I. OF P. CNTR. OF SALES OF AGRIC. EST.	Y regressed on ControlSalesAgricEstate Valid N (listwise)
N	Statistic	380	363
Range	Statistic	8	6.00
Minimum	Statistic	1	1.00
Maximum	Statistic	9	7.00
Sum	Statistic	1729	1588.00
Mean	Statistic	4.55	4.3747
	Std. Error	.114	.11045
Std. Deviation	Statistic	2.213	2.10435
Variance	Statistic	4.897	4.428
Skewness	Statistic	-.171	-.234
	Std. Error	.125	.128
Kurtosis	Statistic	-1.148	-1.267
	Std. Error	.250	.255

Distributions with or without missing?

- What difference do the 17 missing observations make in the
  - Quantile-Normal plot?
  - Box plot?







Data collection and data quality I

- Questions – techniques for asking questions will not be discussed
- Sample
  - From sampling to final data matrix: selection of cases, refusing to participate, and missing answers on questions
- Variables: Data on cases collected as variable values for each case
- Statistics: Data on samples collected as statistics (Norwegian: “observatorer” where values are estimated for each sample
- Statistics is also the science of assessing the quality of each statistic

Data collection and data quality II

- What is important for the quality of the data?
  - Validity of questions asked and reliability of the procedures used.
  - Selection bias: A possible causal link between missing observations and the topic studied
- What can be done if data are faulty?
  - Not much!

Writing up a model

- Defining the elements of the model
  - Variables, error term, population, and sample
- Defining the relations among the elements of the model
  - Sampling procedure, time sequence of the events and observations, the functions that links the elements into an equation
- Specification of the assumptions stipulated to be true in order to use a particular method of estimation
  - Relationship to substance theory (specification requirement)
  - Distributional characteristics of the error term

Elements of a model

- Population (who or what are we interested in?)
- Sample (simple random sample or exact specification of how each case came into the sample)
- Variables (characteristics of cases relevant to the questions we are investigating)
- Error terms (the sum of impacts from all other causes than those explicitly included)

Relations among elements of a model

- Sampling: biased sample?
- Time sequence of events and observations (important to aid causal theory)
- Co-variation (genuine vs spurious co-variation)
  - Conclusions about causal impacts require genuine co-variation
- Equations and functions

Bivariat Regression: Modelling a population

- $Y_i = \beta_0 + \beta_1 x_{1i} + \varepsilon_i$
- i=1,...,n n = # cases in the population
- Y and X must be defined unambiguously, and Y must be interval scale (or ratio scale) in ordinary regression (OLS regression)

Bivariat Regression: Modelling a sample

- $Y_i = b_0 + b_1 x_{1i} + e_i$
- i=1,...,n n = # cases in the sample
- e_i is usually called the residual (mot the error term as in the population model)
- Y and X must be defined unambiguously, and Y must be interval scale (or ratio scale) in ordinary regression (OLS regression)

An example of a bad regression

- The example following contains a series of errors. If you present such a regression in your term paper you will fail
- Your task is to identify the errors as quickly as possible and then never do the same
- Clue: look again at the distributions of the variables above

Importance of public control of sales of agric. Estates

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.047(a)	.002	.000	2.213

a Predictors: (Constant), NUMBER OF DEKAR LAND OWNED

Importance of public control of sales of agric. Estates

ANOVA(b)

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	4.145	1	4.145	.846	.358(a)
	Residual	1851.905	378	4.899		
	Total	1856.050	379			

a Predictors: (Constant), NUMBER OF DEKAR LAND OWNED
b Dependent Variable: I. OF P. CNTR. OF SALES OF AGRIC. EST.

Importance of public control of sales of agric. Estates

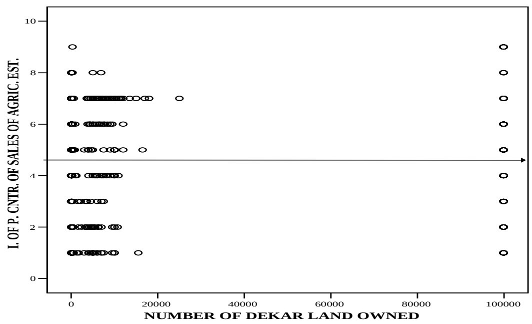
Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	4.610	.131		35.233	.000
	NUMBER OF DEKAR LAND OWNED	.000	.000	-.047	-.920	.358

a Dependent Variable: I. OF P. CNTR. OF SALES OF AGRIC. EST.

Scatterplot

Scatterplot with regression line



Assumptions needed for the use of OLS to estimate a regression model

OLS: ordinary least squares (minste kvadrat metoden)

Requirements for OLS estimation of a regression model can shortly be summed up as

- We assume that the linear model is correct (true) with independent, and identical normally distributed error terms (”normal i.i.d. errors”)

Estimation method: OLS

- Model $Y_i = b_0 + b_1 x_{1i} + e_i$

The observed error (the residual) is

- $e_i = (Y_i - b_0 - b_1 x_{1i})$

Squared and summed residual

- $\Sigma_i(e_i)^2 = \Sigma_i (Y_i - b_0 - b_1 x_{1i})^2$

Find b_0 and b_1 that minimizes the squared sum

Relationship sample - population (1)

- A new mathematical operator: E[¤] meaning the expected value of [¤] where ¤ stands for some expression containig at least one variable or unknown parameter, e.g.
- $E[Y_i] = E[b_0 + b_1 x_{1i} + e_i]$

  $= \beta_0 + \beta_1 x_{1i}$
- Note in particular that in our model
  - $E[b_0] = \beta_0$
  - $E[b_1] = \beta_1$
  - $E[e_i] = \varepsilon_i$

Relationship sample – population (2)

- Relationship sample - population is determined by the characteristics that the error term has been given in the sampling and observation procedure
- In a simple random sample with complete observation

$E[\ \varepsilon_i\ ] = 0$ for all i, and

$var\ [\varepsilon_i] = \sigma^2$ for all i

NB: var(¤) is a new mathematical operator meaning ”the procedure that will find the variance of some algebraic expression ”¤”

Complete observation

- Make it possible to make a completely specified model. This means that all variables that causally affects the phenomenon we study (Y) have been observed, and are included in the model equation
- This is practically impossible. Therefore the error term will include also unobserved factors affecting (Y)

Testing hypotheses I

	In reality H_0 is true	In reality H_0 is untrue
We conclude that H_0 is true	Our method gives the correct answer with probability $1 - \alpha$	Error of type II (probability $1 - \beta$)
We conclude that H_0 is untrue	Error of type I The **test level** α is the probability of errors of type I	Our method gives the correct answer with probability β (= power of the test)

Testing hypotheses II

- A test is always constructed based on the assumption that H_0 is true
- The construction leads to a
  - **Test statistic**
- The test statistic is constructed so that is has a known probability distribution, usually called a
  - **sampling distribution**

Testing hypotheses III

- It is easier to construct tests based on the assumption that it is true that a particular test statistic is zero, [H_0 stating that a parameter is 0], than any particular other value
- In regression this means that we assume a particular parameter $\beta = 0$ in order to evaluate how large the probability is for this to be true given the sample we have observed

The p-value of a test

- The p-value of a test gives the estimated probability for observing the values we have in our sample or values that are even more in accord with a conclusion that **H_0 is untrue**; assuming that our sample is a simple random sample from the population where H_0 in reality is true
- Very low p-values suggest that we cannot believe that H_0 is true. We conclude that $\beta \neq 0$

T-test and F-test

- Sums of squares
  - TSS = ESS + RSS
  - RSS = $\Sigma_i(e_i)^2 = \Sigma_i(Y_i - \hat{Y}_i)^2$ distance observed- estimated value
  - ESS = $\Sigma_i(\hat{Y}_i - \bar{Y})^2$ distance estimated value - mean
  - TSS = $\Sigma_i(Y_i - \bar{Y})^2$ distance observed value – mean
- Test statistic
  - **t = (b - β)/ SE_b** SE = standard error
  - **F = [ESS/(K-1)]/[RSS/(n-K)]** K = number of model parameters

Confidence interval for β

- Picking a t_α- value from the table of the t-distribution with n-K degrees of freedom makes the interval

  $< b - t_\alpha(SE_b) , b + t_\alpha(SE_b) >$

  into a two-tailed test giving a probability of α for committing error of type I
- This means that $b - t_\alpha(SE_b) \leq \beta \leq b + t_\alpha(SE_b)$ with probability 1 – α

Coefficient of determination

Coefficient of determination:

- R^2 = ESS/TSS = $\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2 / \sum_{i=1}^{n}(Y_i - \bar{Y})^2$
  - Tells us how large a fraction of the variation around the mean we can ”explain by” (attribute to) the variables included in the regression ($\hat{Y}_i$ = predicted y)
- In bi-variate regression the coefficient of determination equals the coefficient of correlation: $r_{yu}^2 = s_{yu} / s_y s_u$
- Co-variance: $s_{yu} = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})\left(U_i - \bar{U}\right)$

Detecting problems in a regression

- Take a second look at the example presented above where
  - Y = IMPORTANCE OF PUBLIC CONTROL OF SALES OF AGRICULURAL ESTATES
  - **X =** NUMBER OF DEKAR LAND OWNED
  - $Y_i = b_0 + b_1 x_{1i} + e_i$

What was the problem in this example?

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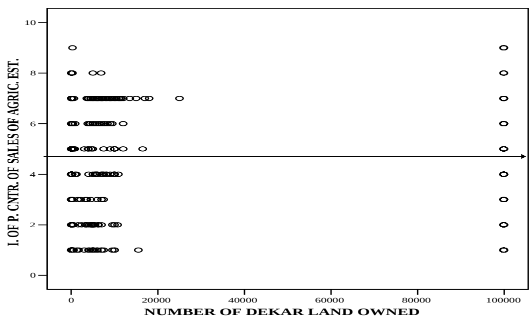
What is wrong in this scatter plot with regression line?



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In general: what can possibly cause problems?

- Omitted variables (specification error)
- Non-linear relationships (specification error)
- Non-constant error term (heteroskedastisitet)
- Correlation among error terms (autocorrelation)
- Non-normal error terms

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Problems also from

- High correlations among included variables (multicollinearity)
- High correlation between an included and an excluded variable (spurious correlation in the model)
- Cases with high influence
- Measurement errors

Non-normal errors:

- Regression **DO NOT need assumptions about the distribution of variables**
- But to test hypotheses about the parameters we need to assume thet the **error terms are normally distributed** with the same mean and variance
- **If the model is correct** (true) and n (number of cases) is large the central limit theorem demonstrates that the error terms approach the normal distribution
- **But usually a model will be erroneously or incompletely specified**. Hence we need to inspect and test residuals (observed error term) to see if they actually are normally distributed

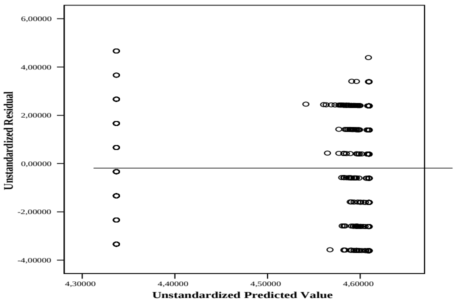
Residual analysis

- This is the most important starting point for diagnosing a regression analysis

Useful tools:

- Scatter plot
- Plot of residual against predicted value
- Histogram
- Box plot
- Symmetry plot
- Quantil-normal plot

What went wrong?
(1) residual-predicted value plot



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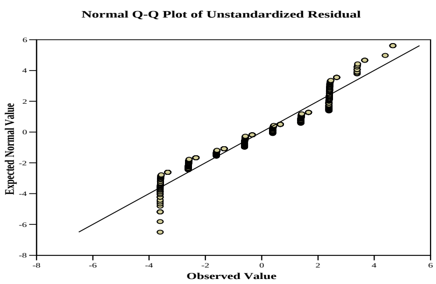
What went wrong?
(1) normal-quantile plot



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Power transformations

May solve problems related to

- Curvilinearity in the model
- Outliers
- Influential cases
- Non-constant variance of the error term (heteroscedasticity)
- Non-normal error term

NB: Power transformations are used to solve a problem. If you do not have a problem do not solve it.

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Power transformations (see H:17-22)

Y* : read "transformed Y"
(transforming Y to Y*)

- $Y^* = Y^q \quad q>0$
- $Y^* = \ln[Y] \quad q=0$
- $Y^* = -[Y^q] \quad q<0$

Inverse transformation
(transforming Y* to Y)

- $Y = [Y^*]^{1/q} \quad q>0$
- $Y = \exp[Y^*] \quad q=0$
- $Y = [-Y^*]^{1/q} \quad q<0$

Power transformations: consequences

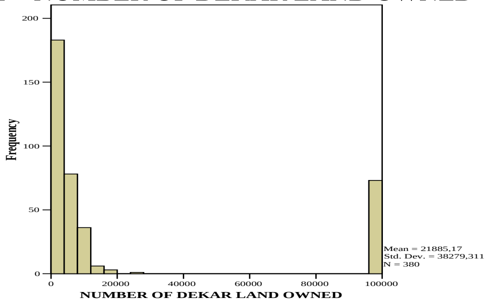
- $X^* = X^q$
  - q > 1 increases the weight of the right hand tail relative to the left hand tail
  - q = 1 produces identity
  - q < 1 reduces the weight of the right hand tail relative to the left hand tail
- If $Y^* = \ln(Y)$ the regression coefficient of an interval scale variable X can be interpreted as % change in Y per unit change in X

  E.g. if $\ln(Y) = b_0 + b_1 x + e$

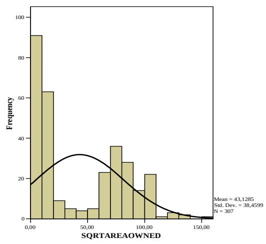
  b_1 can be interpreted as % change in Y pr unit change in X

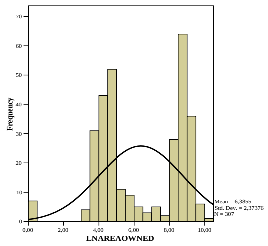
Point of departure

X = NUMBER OF DEKAR LAND OWNED



Power transformed
X = NUMBER OF DEKAR LAND OWNED





SQRT=square root of areaowned – LN= natural logarithm of (areaowned+1)

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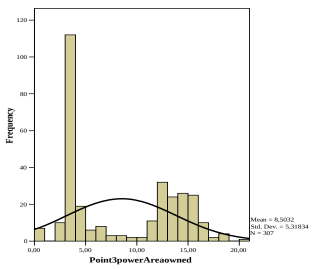
Power transformed
X = NUMBER OF DEKAR LAND OWNED



Point3power = 0,3 power of areaowned

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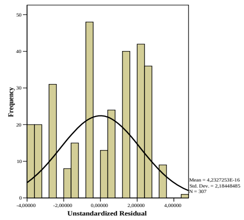
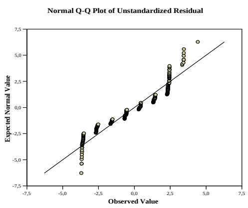
Does power transformation help?





0.3 power-transformation gives lighter tails and no outliers

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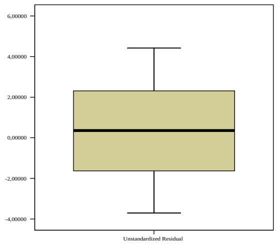
Box plot of the residual shows approximate symmetry and no outliers



Curvilinear regression

- The example above used the variable ”Point3powerAreaowned”, or 0.3 power of number of dekar land owned:
- Point3powerAreaowned = (NUMBER OF DEKAR LAND OWNED)$^{0.3}$

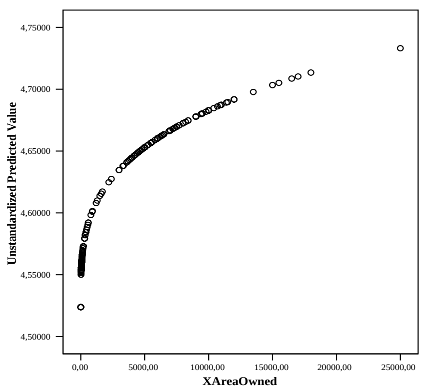
The model estimated is thus

$y_i = b_0 + b_1 (x_i) + e_i$

$y_i = b_0 + b_1 (\text{Point3powerAreaowned}_i) + e_i$

$\hat{y}_i = 4.524 + 0.010 * (\text{NUMBER OF DEKAR LAND OWNED}_i)^{0.3}$

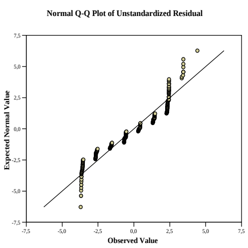
Use of power transformed variables means that the regression is curvilinear

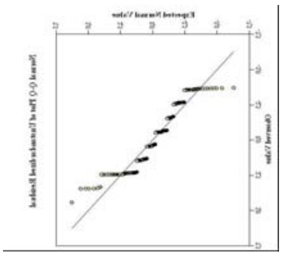


Summary

- In bivariate regression the OLS method finds the ”best” LINE or CURVE in a two dimensional scatter plot
- Scatter-plot and analysis of residuals are tools for diagnosing problems in the regression
- Transformations are a general tool helping to mitigate several types of problems, such as
  - Curvilinearity
  - Heteroscedasticity
  - Non-normal distributions of residuals
  - Case with too high influence
- Regression with transformed variables are always curvilinear. Results can most easily be interpreted by means of graphs

SPSS printout vs the book (see p16)





Reading printout from SPSS (1)

Descriptive Statistics	Mean	Std. Deviation[1]	N[2]
I. OF P. CNTR. OF SALES OF AGRIC. EST.	4.61	2.185	307
Point3powerAreaowned	8.5032	5.31834	307

Model	R	R Square[3]	Adjusted R Square[4]	Std. Error of the Estimate[5]	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.024(a)	.001	-.003	2.188	.001	.182	1	305	.670

a Predictors: (Constant), Point3powerAreaowned
b Dependent Variable: I. OF P. CNTR. OF SALES OF AGRIC. EST.

Footnotes to the table above (1)

1. Standard deviation of the mean
2. Number of cases used in the analysis
3. Coefficient of determination
4. The adjusted coefficient of determination (see Hamilton page 41)
5. Standard deviation of the residual

   s_e = SQRT (RSS/(n-K)),

   where SQRT (*) = square root of (*)

Reading printout from SPSS (2)

Model		Sum of Squares[3]	df	Mean Square	F[1]	Sig.[2]
1	Regression	.870	1	.870	.182	.670(a)
	Residual	1460.224	305	4.788		
	Total	1461.094	306			

- Sums of squares: TSS = ESS + RSS
- RSS = $\Sigma_i(e_i)^2 = \Sigma_i(Y_i - \hat{Y}_i)^2$: sum of squared (distance observed – estimated value)
- Mean Square = RSS / df For RSS it is known that df=n-K

  K equals number of parameters estimated in the model (b_0 og b_1)

  Here we have n=307 and K=2, hence Df = 305

Footnotes to the table above (2)

1. F-statistic for the null hypothesis $\beta_1 = 0$ (see Hamilton p45)
2. p-value of the F-statistic: the probability of finding a F-value this large or larger assuming that the null hypothesis is correct
3. Sums of squares
   1. TSS = ESS + RSS
   2. RSS = $\Sigma_i(e_i)^2 = \Sigma_i(Y_i - \hat{Y}_i)^2$ distance observed value – estimated value
   3. ESS = $\Sigma_i(\hat{Y}_i - \bar{Y})^2$ distance estimated value – mean
   4. TSS = $\Sigma_i(Y_i - \bar{Y})^2$ distance observed value – mean

Reading printout from SPSS (3)

Model		Unstandardized Coefficients		Standardized Coefficients			95% Confidence Interval for B	
		B[1]	Std. Error[2]	Beta[3]	t[4]	Sig.[5]	Lower Bound	Upper Bound
1	(Constant)	4.524	.236		19.187	.000	4.060	4.988
	Point3-powerA rea-owned	.010	.024	.024	.426	.670	-.036	.056

Footnotes to the table above (3)

1. Estimates of the regression coefficients b_0 og b_1
2. Standard error of the estimates of b_0 og b_1
3. Standardized regression coefficients: $b_1^{st} = b_1*(s_x/s_y)$ see Hamilton pp38-40
4. t-statistic for the null hypothesis $beta_1 = 0$ (see Hamilton p44)
5. p-value of the t-statistic: the probability of finding a t-value this large or larger assuming that the null hypothesis is correct